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An Elementary Treatise on Calculus. By WILLIAM S. FRANKLIN, BARRY MACNUTT and ROLLIN L. CHARLES. Published by the authors, South Bethlehem, Pa., 1913. x+253+41 pages. $2.00.

This book, styled in the subtitle "a textbook for colleges and technical schools," is a noteworthy contribution to the texts carrying out the "Perry movement" in this country. The outstanding feature of the book is expressed in the preface in the purpose of the authors "to lead the student to a clear understanding of principles" and "to develop the subject as simply and as directly as possible." And yet it is exactly at this point that the reviewer's chief criticism is raised; this is in regard to the introduction of infinitesimals as indicated by the following quotation from page 23 of the text:

> "In many cases, however, it is desirable to find the limiting relation between Δy and Δx when Δx approaches zero *without deriving an expression for* $\Delta y/\Delta x$. For example let $y = ax^3$. Then
>
> $$\Delta y = 3ax^2 \cdot \Delta x + 3ax(\Delta x)^2 + a(\Delta x)^3. \tag{1}$$
>
> Both members of this equation approach zero when Δx approaches zero, and it would therefore seem rather difficult to determine the limiting relation between Δy and Δx (when Δx approaches zero) from this equation as it stands. But when Δx is made as small as you please, then $(\Delta x)^2$ is infinitely smaller than Δx, and $(\Delta x)^3$ is infinitely smaller than $(\Delta x)^2$. For example, let Δx be a millionth of a unit, ... Therefore the terms $3ax(\Delta x)^2$ and $a(\Delta x)^3$ become more and more nearly negligible in comparison with $3ax^2\Delta x$ as Δx grows smaller and smaller in equation (1). The *limiting relation* between Δy and Δx may be found by writing dx and dy for Δx and Δy to indicate that we have proceeded to the limit, and by dropping every term which contains the square or any higher power of Δx (or the square or any higher power of Δy). This gives
>
> $$dy = 3ax^2 \cdot dx. \tag{2}$$
>
> In this equation dy and dx are as small as you please and they are called *infinitesimals;* . . . ."

In their preface the authors disclaim any intention of taking sides in the old controversy about the method of limits and that of infinitesimals, and they acknowledge there that the treatment here given is fallacious. The question then arises fairly and squarely whether they as textbook makers are justified, on grounds of simplicity or practical purposes or for any such reason, in using an analysis so inexact in the presentation of the fundamental ideas of the calculus. The writer is frank to say that in his judgment they are not justified. Beginners in calculus grasp the technique of differentiating and integrating rather than the analytic work involved in the proofs of the formulas and in the opening discussions, which latter should give *exact* notions of the calculus; and it surely does not sharpen these immature minds to read a text which with only a slight explanation comes at once to such steps as are given in the last two sentences quoted. The development much to be preferred *for all students* is that shown by the following excerpts from the 1912 "Syllabus of Mathematics" compiled by the committee on the teaching of mathematics to students of engineering under the direction of the Society for the Promotion of Engineering Education:[1]

> "The ratio $\Delta y/\Delta x$ may be called the *average rate of change of the function during the interval from* $x = x_0$ *to* $x = x_0 + \Delta x$. (Geometrically, $\Delta y/\Delta x$ is the slope of the secant.) Now let Δx

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[1] Copies can be obtained from the Secretary, Professor H. H. Norris, Ithaca, N. Y.

> approach zero. . . . Then *the ratio $\Delta y/\Delta x$ will in general approach a definite limit, and this limit is called the actual rate of change at the point* $x = x_0$ (geometrically, the limit of $\Delta y/\Delta x$ is the slope of the tangent).
>
> "The *rate of change* of a function $y = f(x)$ at any point, or the *slope of the curve* at that point, is called the *derivative* of the function at that point, and is denoted by $f'(x)$, ... The value that Δy *would have* if the curve coincided with its tangent is called the *differential* of y. . . . The use of differentials gives us a *new notation for the derivative,* $f'(x) = dy/dx$."

It should be said that in the last statement quoted it is implied, but was unfortunately not stated clearly, that the right-hand member is the quotient of two differentials.

To return to the text under review, the first chapter—a general survey of differential and integral calculus—is ably, because concretely, developed, presenting successively the notions of rate of change, formal derivative, differential, functions which have the same derivative, work required to stretch a spring, area under a parabola, differential equations, the definitions of indefinite integral and definite integral, with the symbol for the last. Inasmuch as about fifty problems are listed in this introductory chapter, it will be seen that the basic principles of both branches of the calculus need to be fairly mastered at the outset of the course. It would be a fair question to raise whether it is not crowding the student beyond his ability to give him all these ideas in one chapter, when, as we all know, the single abstract notion of differentiation is of such difficulty to many in our classes that it is only slowly grasped—*actually grasped* and handled as a working tool—by the beginner. If on the other hand it is not meant that these new notions should all be mastered at once, why present them all in one chapter? A weakness in Art. 7 may be noted here. Where the *temperature gradient* is defined, the argument is made that the temperature T along a rod must vary *continuously,* because "otherwise the temperature gradient of T at a point would be unthinkable." Only if the students were already acquainted with the idea of gradient, would this argument be convincing.

Chapter II develops in the usual fashion the standard formulas for differentiation. These are referred to in the next chapter, on "Integration," as rules for integration also. In addition, applications of these are made to velocity and acceleration, harmonic motion, curvature, and, as examples of geometric differentiation, to the acceleration of a particle moving in a circular orbit, and to the inward force per unit of length of a barrel hoop, with a commendable emphasis throughout on the proper units in which certain distances, velocities, accelerations, etc., should be expressed. See, for example, problem 1, page 36, and problem 10, page 58.

Still further evidence of the concrete character of the text is given by the very novel introduction in Chapter III of the planimeter to explain the idea of integration, as well as by reference to the cyclometer, electricity meter, and car "speed-time" diagram. Indeed, so completely is the emphasis upon the practical side that the whole technique of integration is disposed of in four pages, by referring to integration by inspection, by the aid of four of the simplest of the usual rules for integration, and, beyond this, by the use of the fairly complete integral tables given at the end of the book. Of course, practice in these is given through a classified collection of some sixty-nine problems. In order to present an unbroken discussion of the principles, the problems for the second and third chapters are relegated to an appendix.

Chapter IV treats of partial differentiation and integration. Partial derivatives are presented by a clever use of the device of a hill built upon the xy-plane, this device being used consistently throughout the explanation of "gradient in two dimensions" and volumes and surfaces as double integrations.

Chapter V, on "Miscellaneous Applications," combines the topics of probable errors of derived results (applying partial derivatives), maximum and minimum values, problem of the bent beam, average value of a function, center of gravity and of pressure, and moment of inertia. The last includes the proof of the principle of parallel axes, but supplies only two problems to which this may be applied; the abbreviated measure of practice problems accorded to this and other important principles is evidently to be explained by the unusually extensive range of subjects transferrred to this course from the second and even third course in calculus, as traditionally arranged.

In consonance with the general method adopted for the book, Chapter VI gives a "plausible" proof of Maclaurin's theorem and from this derives Taylor's theorem, but evinces a desire to make no distinction between the two. The reviewer cannot but regard this as unwise, the former being universally used as a separate form, even though it be a special case of the latter. This chapter presents also Demoivre's theorem, hyperbolic functions, and Euler's equations which express $\sin x$ and $\cos x$ in exponential form. Except for illustrating the utility of these equations for the electrical engineer, the integration of $\sin mx \cos nx$ might better be carried out by means of the substitution:

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$$

Chapter VII describes certain ordinary differential equations, presents several well chosen illustrations from physics of the "principle of superposition," represented mathematically in the solutions of linear differential equations with constant coefficients; and, under the figure of such familiar examples as the starting and stopping of a boat and the undamped and damped oscillations of a weighted coiled spring, points to the fundamental problems of the theory of alternating currents.

Chapter VIII treats skilfully and clearly the partial differential equation of wave motion, applying it by the method of infinitesimals to the vibrations of a stretched string, and leading of course to an outline treatment of Fourier's sine and cosine series, the proofs as to the convergence and equivalence of the series being waived. That the gist of Fourier's series should be condensed into eight brief pages and in a manner that would seem to appeal readily to the *practical* thinker, is truly noteworthy; it is a happy example of "the new mathematics."

The final Chapter IX startles the reader still more with as simple and sensible a treatment of vector analysis as the writer remembers ever to have seen. True, much that is in the chapter must be mastered through the instructor's exposition, but we must make proper recognition when we meet with a treatment of vectors that continually keeps before the student the knowledge that vectors *can be used* and that makes the theorem that the curl of a gradient is zero appear as a matter of common sense (even to a pure mathematician). This is done by means of a well-explained example of a curl. The chapter closes with suggestive, rather than complete, proofs of Stokes's theorem and the formulas for small displacements, with incidental discussion of vector fields and potential.

An appendix provides a careful but broad selection and description of texts to be recommended for the student's further study in mathematics and mathematical physics.

William DeW. Cairns.

PROBLEMS AND SOLUTIONS.

B. F. Finkel, Chairman of the Committee.

PROBLEMS FOR SOLUTION.

ALGEBRA.

398. Proposed by R. D. CARMICHAEL, Indiana University.

In the equation $x^3 + \alpha x + \beta = 0$, α is an integer divisible by p^2 and β is an integer divisible by p, p being a prime number. Prove that β is divisible by p^3 if the equation is reducible.

399. Proposed by W. H. BUSSEY, University of Minnesota.

A borrows from B \$1,500 and pays back \$34 a month for 63 months. If the last payment closes the account, what rate of interest has A been paying?

400. Proposed by C. N. SCHMALL, New York City.

Sum the series

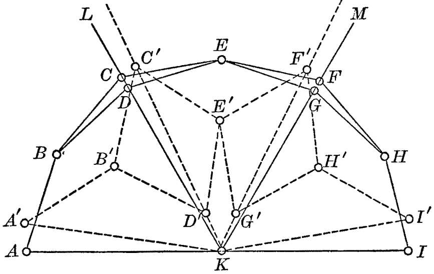
$$1 + 2x + 3x^2 + 4x^3 + \cdots$$

(Bromwich, *Infinite Series*, p. 129, ex. 1.)

GEOMETRY.

427. Proposed by F. CAJORI, Colorado College.

In S. Gross's linkage for trisection of angles, shown in the figure (KL' and KM' being the



trisectors of $A'KI'$), C is fixed on KL, also F on KM; at starting, C and D coincide, also F and G;